



Inquiry-based learning and epistemic beliefs as foundations of sustainable mathematical problem-solving in secondary education

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Abstract

This study examines the effects of inquiry-based learning (IBL) and epistemic beliefs on senior high school students' perceived mathematical problem-solving competence within the context of sustainable mathematics education. A quantitative cross-sectional design was employed, and data were collected from 395 senior high school students using a structured questionnaire. Exploratory factor analysis and reliability analyses were conducted using SPSS version 27, while confirmatory factor analysis and structural equation modeling were performed using AMOS version 23. The findings showed that IBL had a significant positive effect on students' perceived problem-solving competence (PPSC) and epistemic beliefs. Epistemic beliefs also significantly and positively predicted PPSC. However, the interaction between IBL and epistemic beliefs was not statistically significant, indicating that epistemic beliefs did not moderate the relationship between IBL and PPSC. These findings suggest that IBL and epistemic beliefs make independent contributions to students' perceived mathematical problem-solving competence. From a sustainable mathematics education perspective, the results highlight the importance of instructional practices that promote inquiry, reasoning, reflection, and more sophisticated beliefs about mathematical knowledge in supporting long-term problem-solving development.

Keywords: Epistemic beliefs; Inquiry-based learning; Mathematics education; Perceived problem-solving competence; Senior high school

1. Introduction

Developing problem-solving ability is likely the most important mathematics education objective (Davor, Asare, & Abanamu, 2026). Making informed decisions, modeling, and applying mathematical concepts to novel situations is central to being successful in school and later in life (Szabo et al., 2020; Zhu et al., 2024; Chen, 2025). IBL has been a successful pedagogy of the last decade that promotes such higher-order thinking abilities. It provides students with the autonomy to explore problems, construct their solutions, and document their thinking through active engagement instead of memorization (Gradini et al., 2025). Directed exploration in inquiry learning helps students develop a deeper conceptual understanding of mathematics and enhances their capacity to use mathematical knowledge in novel situations. Notwithstanding such advantages being identified, research still shows that the effectiveness of inquiry learning is inconsistent for students. Although some students benefit from inquiry activities and demonstrate improved PPSC, others are unable to carry out IBL without extensive teacher guidance (Hamenu & Davor, 2026; Schiefer et al., 2022). The difference shows that students' beliefs and epistemical orientations dictate how they learn through IBL instruction. One of the key constructs in this regard is mathematical epistemic belief, which explains how learners view the nature of mathematics and how it is acquired. Students who view mathematics as a reasoning, reflection, and discovery approach to problems are more creative and persistent. In contrast, individuals who perceive mathematics as a prescriptive instruction of rules and formulas limit their involvement to procedural learning (Kersting et al., 2023; Strat et al., 2024).

Epistemic beliefs can therefore affect how effectively students learn from inquiry-based instruction. Where students believe that mathematics knowledge is concerned with constructing meaning and critiquing reasoning, they would embrace the investigative element of inquiry

learning. Students with less developed epistemic beliefs will find inquiry activities confusing or unfruitful, while having the potential to limit the effectiveness of the approach. However, some studies have individually concentrated on problem-solving and IBL; few have examined the role epistemic beliefs play in bridging the two (Muis & Chen, 2025). This is most true for secondary school levels in developing countries like Ghana, where mathematics curricula are being reformulated with greater emphasis on inquiry-based practices (Alkış Küçükaydın et al., 2025; Davor et al., 2025). Policymakers and teachers must understand how beliefs of students regarding mathematical knowledge impact the efficacy of IBL. It can guide educators in the design of learning experiences that can be helpful to various belief systems of students and provide appropriate scaffolding to those who are struggling with open-ended tasks. It is for this reason that the present research investigates how students' mathematics epistemic beliefs moderate the impact of IBL on PPSC for senior high school students. Beyond short-term academic outcomes, this study is positioned within the broader framework of sustainable mathematics education, which emphasizes the development of durable reasoning skills, adaptive problem-solving abilities, and epistemic growth that extend beyond immediate classroom performance. IBL and epistemic beliefs are therefore examined not only as predictors of perceived competence but as foundational elements for long-term learner development and deep mathematical understanding (Szabo et al., 2020; Zhu et al., 2024).

IBL has been widely promoted in mathematics education as a means of improving students' understanding and problem-solving ability. However, not all students benefit equally from this approach. In many classrooms, some learners actively engage in inquiry-based lessons, while others remain passive and demonstrate limited improvement in their PPSC (Kersting et al., 2023; Szabo et al., 2020). This suggests that individual learner characteristics, particularly epistemic beliefs, may influence how students respond to inquiry-based instruction. Epistemic beliefs shape how learners interpret knowledge, engage with tasks, and evaluate solutions (Muis & Chen, 2025; Xu et al., 2024). Despite this, there remains limited empirical evidence examining whether epistemic beliefs influence the effectiveness of IBL in shaping students' PPSC, particularly in secondary school contexts within developing countries.

The contribution of this study lies in addressing the limited empirical understanding of how students' epistemic beliefs influence the relationship between IBL and PPSC in Ghanaian senior high schools. Specifically, the study examines whether epistemic beliefs strengthen or weaken the effectiveness of IBL in shaping students' perceived competence in solving mathematical problems.

2. Literature Review and Development of Hypotheses

2.1. IBL and PPSC

IBL is emerging as a leading pedagogical approach to strengthening conceptual comprehension, reasoning, and mathematical problem-solving. It engages students in questioning, investigating math relations, and evolving explanations through inquiry and reflection (Schiefer et al., 2022). Rather than memorizing algorithms and formulas, students in inquiry classrooms construct their own knowledge through inquiry and collaboration. This learn-by-doing approach acquires procedural, along with conceptual knowledge necessary for problem-solving (Barmby et al., 2025; Lee et al., 2024). Hwang et al. (2024) ascertained that students instructed in inquiry-based mathematics acquired more development in modeling and reasoning abilities than the students instructed using conventional strategies. Apart from that, Asante et al. (2025) indicated that the use of inquiry assignments within Ghanaian classrooms enhanced the creativity and perseverance of students when solving equations. The findings indicate that IBL enhances mathematical concepts learning deeply and encourages students to transfer what they have learned to new situations. Nonetheless, its effect may tend to differ depending on individual learners' characteristics, including beliefs, motivation, and learning styles. Inquiry learning, therefore requires students who can engage with math problems in active and reflective ways.

2.2. Epistemic Beliefs in Mathematics

Epistemic beliefs have been conceptualized by researchers as individuals' assumptions about the nature of knowledge and knowing. These epistemic mathematical beliefs influence students' conception of mathematical knowledge, justification for arguments, and how they engage in learning activities (Xu et al., 2024). Students with sophisticated epistemic beliefs view mathematics as a dynamic and changing field based on justification and reasoning. Such students find meaning, monitor their understanding, and engage actively in activities. In addition, naïve epistemic beliefs, students are likely to perceive mathematics as a set of hard and fast rules and facts memorized and strictly applied (Depaepe et al., 2016; Gupta & Elby, 2011; Muis & Chen, 2025). Epistemic beliefs were found in a range of studies to affect math learning and problem-solving among students. Similarly, Tang (2025) found that epistemic sophistication generates more engagement and more conceptual understanding of mathematics. Epistemic beliefs are also supportive of inquiry classrooms since learners in an inquiry classroom need to justify the explanations, interpret the relationships, and reach conclusions independently. Therefore, students' epistemic beliefs can influence what they learn from inquiry instruction and how they apply inquiry experiences to effective problem-solving abilities.

- H1) IBL has a significant positive effect on students' PPSC.
- H2) IBL has a significant positive effect on students' epistemic beliefs.
- H3) Students' epistemic beliefs have a significant positive effect on PPSC.
- H4) Students' epistemic beliefs moderate the relationship between IBL and PPSC.

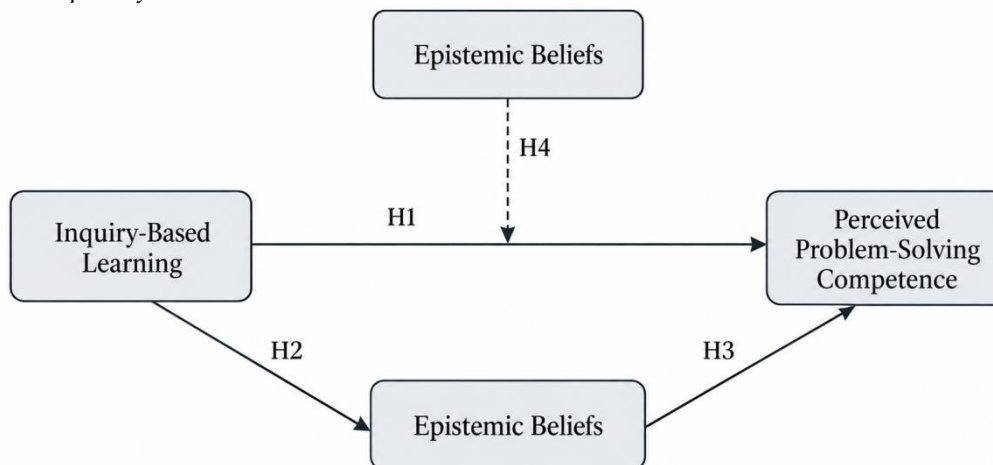
3. Theoretical and Conceptual Framework

This study is grounded in constructivist learning theory and epistemic cognition theory, which together help explain how inquiry-based instruction and students' epistemic beliefs relate to perceived mathematical problem-solving competence. Constructivist perspectives emphasize that learners actively build knowledge through interaction, exploration, and reflection rather than passively receiving information (Palincsar, 2012; Vygotsky, 1978). In mathematics education, this perspective suggests that students learn more deeply when they are encouraged to question, test assumptions, justify their reasoning, and connect new ideas with prior knowledge. IBL reflects this constructivist orientation by positioning learners as active participants in the construction of mathematical knowledge.

Constructivist learning environments such as IBL also require learners to manage uncertainty, evaluate evidence, and reflect on how knowledge is justified. Epistemic beliefs—learners' beliefs about the nature of knowledge and knowing—are therefore relevant to inquiry-oriented learning. Epistemic cognition theory explains how such beliefs can shape learners' task interpretations, use of evidence, and regulation of learning (Hofer, 2016; Hofer & Pintrich, 1997). Students with more sophisticated epistemic beliefs are more likely to view mathematical knowledge as constructed and justified through reasoning, whereas less sophisticated beliefs may frame knowledge as fixed and dependent on authority. In the present study, constructivist theory provides the instructional rationale for IBL, while epistemic cognition theory explains why students' beliefs about knowledge may be associated with their PPSC. Related work on mathematics problem solving and beliefs further supports examining these constructs together (Davor, Asare, & Abanamu, 2026).

Figure 1 presents the conceptual model guiding the study. The model specifies direct paths from IBL to PPSC and epistemic beliefs, as well as a direct path from EB to PPSC. It also tests whether EB moderates the association between IBL and PPSC. Thus, the model examines epistemic beliefs both as an independent predictor of perceived competence and as a potential contextual factor in the IBL–PPSC relationship.

Figure 1
Conceptual framework



Note. Author's construct (2025).

4. Method

4.1. Research Design

This study employed a quantitative cross-sectional design to examine the relationships among IBL, epistemic beliefs, and perceived mathematical problem-solving competence, including the proposed moderating role of epistemic beliefs. Structural equation modeling was used to estimate the hypothesized relationships among the constructs. Because the data were cross-sectional, the estimated paths are interpreted as predictive associations rather than causal effects (Kline, 2023).

4.2. Participants

The population of interest consisted of senior high school students studying mathematics in selected schools in the Ashanti Region of Ghana. A multistage sampling technique was employed. Schools were first stratified according to ownership type (public and private), after which proportional random sampling was used to select students from each stratum. The questionnaires were administered in person to ensure accessibility and completeness of responses. Data were collected over a six-week period from March to April 2025. Of the questionnaires distributed, 395 were properly completed and returned and were included in the final analysis.

4.3. Instruments

A structured questionnaire was used as the primary data-collection instrument. The survey was administered in paper format to ensure accessibility. A cover letter explained the purpose of the study, eligibility for participation, and assurances of anonymity and confidentiality. The questionnaire included demographic items and measures of the three study constructs: IBL, epistemic beliefs, and PPSC. Items were rated on a five-point Likert-type scale ranging from 1 (Strongly Disagree) to 5 (Strongly Agree). The epistemic-belief items were adapted from Katrancı and Şengül (2020) whereas the mathematical problem-solving competence items were adapted from Řičan et al. (2022). A pilot test with 40 students was conducted to identify unclear wording, and the questionnaire was revised on the basis of participant feedback. The dependent variable represents students' self-reported perceptions of their problem-solving competence rather than objectively measured mathematical performance. Self-report measures are commonly used to assess perceived capability, confidence, and strategy use (Podsakoff & Organ, 1986); nevertheless, the findings should be interpreted as perceived competence rather than actual performance.

4.4. Common Method Bias

Because the predictor and outcome variables were measured using self-report questionnaires, procedural and statistical steps were used to reduce and assess common method bias (CMB). Respondents were assured of anonymity and confidentiality to reduce evaluation apprehension and socially desirable responding. The questionnaire also separated predictor and criterion measures into different sections, and the study reports a brief temporal separation between the collection of IBL/EB measures and PPSC. After data collection, Harman's single-factor test was conducted by entering all measurement items into an exploratory factor analysis to determine whether a single factor accounted for most of the covariance. A confirmatory factor analysis including a common latent factor was also used as an additional assessment of method variance. These procedures were interpreted as diagnostic checks rather than proof that CMB was absent (Podsakoff & Organ, 1986).

4.5. Validity and Reliability Analysis

Internal consistency was assessed using Cronbach's alpha. IBL, epistemic beliefs, and PPSC yielded alpha coefficients of .938, .930, and .928, respectively (Table 2). These values exceed the commonly used .70 criterion and indicate satisfactory internal consistency (Hair et al., 2017).

4.5.1. Exploratory factor analysis

Exploratory factor analysis (EFA) was conducted to examine the underlying structure of the measurement items. The Kaiser-Meyer-Olkin (KMO) measure of sampling adequacy was .932, exceeding the minimum criterion of .50, and Bartlett's test of sphericity was significant, $\chi^2(78) = 4811.553, p < .001$. The determinant was 5.763×10^{-5} . Factor extraction and item evaluation resulted in a three-factor solution corresponding to IBL, EB, and PPSC, which together explained 81.48% of the variance. Eight items were removed because of weak or problematic cross-loadings. The retained measurement model comprised four IBL items, four EB items, and five PPSC items.

4.5.2. Confirmatory factor analysis

Confirmatory factor analysis (CFA) was conducted in AMOS (version 23) using the items retained after EFA. All standardized factor loadings exceeded .50. Model fit was evaluated using multiple fit indices following established SEM guidance (Hair et al., 2017). A residual covariance between e6 and e8 was specified in the reported model. Because correlated residuals should be theoretically justified rather than introduced solely to improve fit, this modification should be interpreted cautiously. The CFA results are summarized in Table 2 and the model-fit indices are reported in Table 5.

4.5.3. Discriminant validity analysis

Discriminant validity was assessed using the Fornell-Larcker criterion by comparing the square root of each construct's average variance extracted (AVE) with its correlations with the other constructs. Discriminant validity is supported when the square root of AVE for a construct exceeds its correlations with other constructs (Fornell & Larcker, 1981). As shown in Table 3, the smallest square root of AVE (.852) exceeded the largest inter-construct correlation (.674), supporting discriminant validity. In addition, the AVE values for IBL, EB, and PPSC were all above .50 (see Table 4), supporting convergent validity.

4.5.4. Model fit measures

The measurement model demonstrated acceptable fit across the reported indices. The normed chi-square was $\chi^2/df = 1.592$. The comparative fit index (CFI = .988), Tucker-Lewis index (TLI = .985), normed fit index (NFI = .979), goodness-of-fit index (GFI = .974), standardized root mean square residual (SRMR = .030), and root mean square error of approximation (RMSEA = .022) all indicated

Table 2
Confirmatory Factor Analysis (CFA) and KMO and Bartlett's test analysis

Construct	Items	EFA loadings	CFA loadings	CA	CR
Inquiry-Based learning				.938	.939
	IBL2	.875	.939		
	IBL5	.856	.887		
	IBL6	.870	.877		
Epistemic Beliefs	IBL7	.847	.857		
				.930	.931
	EB1	.821	.934		
	EB3	.781	.910		
Perceived Problem-solving Competence	EB5	.820	.880		
	EB7	.808	.783		
				.928	.929
	PSP3	.859	.916		
KMO and Bartlett's test	PSP4	.835	.888		
	PSP5	.831	.841		
	PSP6	.806	.821		
	PSP7	.796	.787		
TVE				81.480%	
Kaiser-Meyer-Olkin measure of sampling adequacy				.932	
Bartlett's test of sphericity		Approx. Chi-square		4811.553	
		df		78	
		p		< .001	
Determinant				5.763E-5	

Table 3
Discriminant Validity

Variables	IBL	EB	PPSC
IBL	.891		
EB	.655***	.879	
PPSC	.508***	.674***	.852

Note. *** $p < .01$. Diagonal values (square roots of AVE) are shown in bold italics.

Table 4
Convergence validity

Measurement items	AVE
Inquiry-Based Learning (IBL)	0.793
IBL2: My math teacher lets me ask questions and explore new ideas.	
IBL5: I understand math better when I explore and find answers on my own	
IBL6: I try to find out things by myself before the teacher explains them.	
IBL7: We do activities that help us use math to solve real-life problems.	
Epistemic Beliefs (EB)	0.772
EB1: To learn math, I need to understand, not just memorize formulas	
EB3: There can be more than one correct way to solve a math problem	
EB5: It is more important to know why a rule works than just to use it.	
EB7: In math, good reasoning is as important as getting the right answer.	
Perceived Problem-Solving Competence (PPSC)	0.726
PSP3: I can use different methods to solve hard math problems.	
PSP4: I believe I can solve most math problems if I keep trying	
PSP5: I check my answers to make sure they make sense.	
PSP6: I can use what I learn in math to solve new kinds of problems.	
PSP7: I enjoy solving difficult math problems.	

good to excellent fit according to the criteria reported in Table 5. The PClose value was .655. Taken together, these indices indicate that the specified measurement model was consistent with the observed covariance structure.

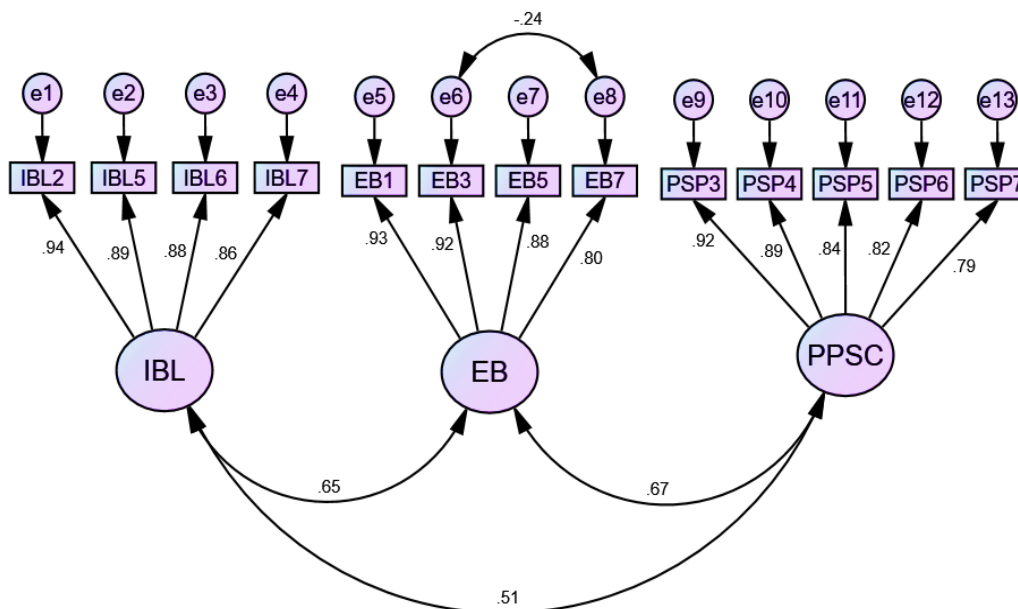
Table 5

The model fit indices

Measures	Estimates	Standard	Interpretation	Source
CMIN	98.704	The smaller the better	-	-
DF	62	The smaller the better	-	-
CMIN/DF	1.592	Between 1 and 3	Good fit	Marsh et al. (2020)
TLI	.985	> 0.95	Excellent	Hair et al. (2017)
CFI	.988	> 0.95	Excellent	Marsh et al. (2020)
NFI	.979	> 0.90	Good fit	Hair et al. (2017)
GFI	.974	> 0.80	Excellent	Hair et al. (2017)
RMSEA	.022	< 0.08	Excellent	Hayduk and Littvay (2012)
PClose	.655	> 0.05	Excellent	Larbi et al. (2026)
SRMR	.030	< 0.08	Good fit	Kline (2023)

Figure 2

Confirmatory factor analysis model



5. Results

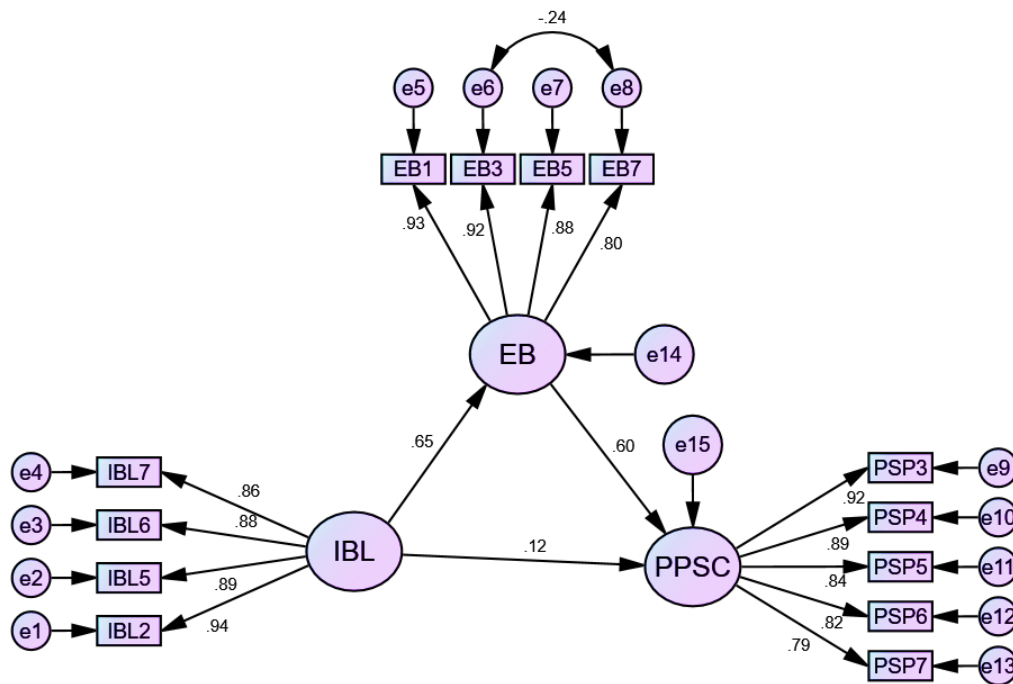
Covariance-based structural equation modeling (CB-SEM) in AMOS (version 23) was used to estimate the hypothesized paths among the study constructs. This approach is appropriate for testing theoretically specified relationships among observed and latent variables (Davor, Boateng, et al., 2026). The path estimates are presented in Table 6. Path analysis are given in Figure 3.

Table 6

Path Summary

Direct Effect	Std. Est.	S.E.	C.R.	p-value
IBL → PPSC	.115	.055	2.100	.036
IBL → EB	.651	.044	14.879	.001
EB → PPSC	.586	.057	10.307	.001

Figure 3
Path analysis



5.1. IBL positively Influences students' PPSC

Hypothesis 1 proposed that IBL would positively predict students' PPSC (PPSC). As shown in Table 6, IBL significantly predicted PPSC ($\beta = .115$, $SE = .055$, $CR = 2.100$, $p = .036$). Thus, H1 was supported, although the standardized effect was small.

5.2. IBL Positively Affects Students' Epistemic Beliefs

Hypothesis 2 proposed that IBL would positively predict students' epistemic belief. The results in Table 6 showed a strong, statistically significant association between IBL and EB ($\beta = .651$, $SE = .044$, $CR = 14.879$, $p = .001$). Thus, H2 was supported.

5.3. Epistemic Beliefs Positively Affect Students' PPSC

Hypothesis 3 proposed that students' epistemic beliefs would positively predict PPSC (PPSC). EB significantly predicted PPSC ($\beta = .586$, $SE = .057$, $CR = 10.307$, $p = .001$), supporting H3.

5.4. Epistemic Beliefs Moderate the Relationship between IBL and Students' PPSC

To test H4, a moderation analysis was conducted to examine whether epistemic beliefs altered the relationship between IBL and students' PPSC. Specifically, an interaction term between IBL and epistemic beliefs was included in the structural model. The results of the moderation analysis are presented in Table 7.

Table 7

Moderating effects

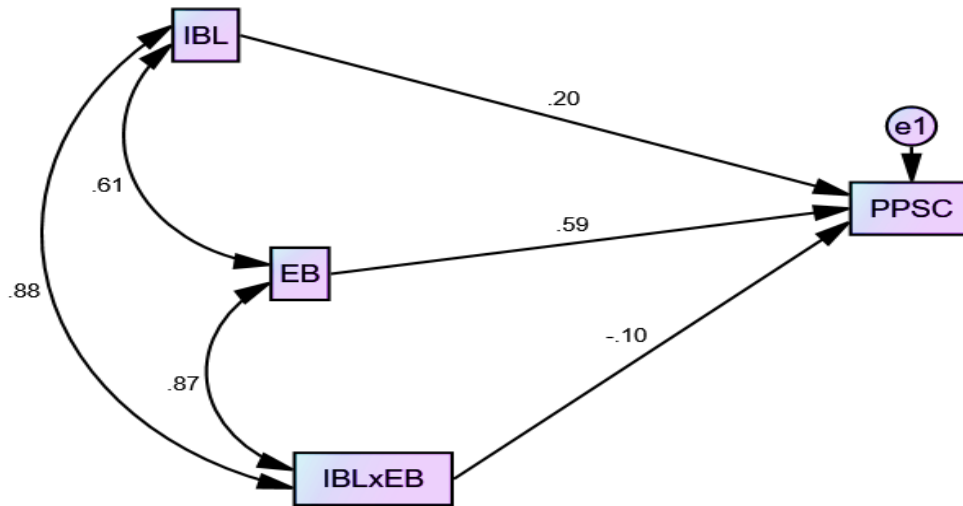
Hypothesis	Moderating effect	Std. Estimate	S. E	CR	p-value
H ₄	IBLxEB→PPSC	-.104	.031	-.549	.583

Note. The moderation statistics in Table 7 require verification against the original AMOS output because $\beta = -.104$, $SE = .031$, $CR = -.549$, and $p = .583$ are not mutually consistent.

The structural moderation model illustrating the interaction between IBL and epistemic beliefs is presented in Figure 4. Figure 5 further illustrates the nature of the interaction by plotting the relationship between IBL and PPSC at different levels of epistemic beliefs.

Figure 4

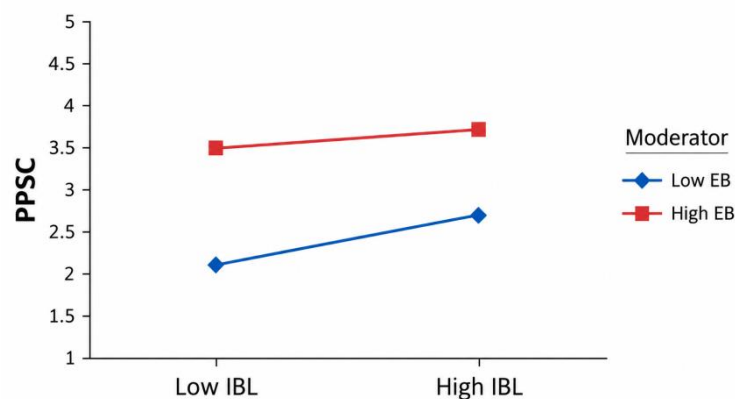
Moderation path



Source. Author's construct (2025).

Figure 5

Two-way interaction plot for the moderation effect of epistemic beliefs



Source. Author's construct (2025).

The results presented in Table 7 and Figure 5 indicate that the reported interaction between IBL and epistemic beliefs was not statistically significant (reported $\beta = -.104$, $p = .583$). Accordingly, H4 was not supported. Although the interaction plot shows slight differences in slopes across levels of epistemic beliefs, these differences are descriptive and should not be interpreted as evidence of moderation. The numerical values reported in Table 7 should be checked against the original AMOS output because the reported standardized estimate, standard error, critical ratio, and p value are not internally consistent.

6. Discussion

The study identified that IBL has a direct and immediate positive impact on PPSC. This is an insightful reflection of the pedagogical impact of student-centered teaching methods in promoting critical thinking and mental stimulation. The positive relationship indicates that students who engage in IBL demonstrate higher PPSC. This was as per the constructivist theory of learning, wherein the knowledge is actively constructed by reflection and exploration (Schiefer et al., 2022; Vygotsky, 1978). The same has also been elaborated by Ayalon and Wilkie (2024) and Chen and Chen (2025), who determined that mathematics instruction based on inquiry enhances the logical

reasoning ability of students through complex problems and the use of conceptual knowledge in an effective way. Asante et al. (2025) mentioned that an inquiry-based classroom develops the self-confidence of students to solve non-routine problems, which results in greater cognitive flexibility and fluency in problem-solving. The present study then provides more empirical support that IBL is an efficient teaching approach to develop higher-order thinking abilities among senior high school students.

Again, the study also found that IBL significantly predicted epistemic beliefs. As a result, involvement with inquiry-based instruction not only is associated with PPSC and with the students' perception of the nature of knowledge and learning (Asiedu Menlah & Boateng, 2025). Students engaged in inquiry lessons have more advanced epistemic beliefs, moving from perceiving knowledge as static to perceiving it as dynamic and constructed through the help of reasoning. This finding aligns with a study conducted by Lammassaari et al. (2024), which highlighted that inquiry lessons lead students to investigate evidence, reason on behalf of claims, and engage in reflective discussion, activities that enhance epistemic development. Similarly, Kaarlöp et al. (2025) discovered that students instructed using inquiry and dialogic approaches are epistemically more sensitive and receptive to multiple perspectives. Thus, the current finding adds support to the argument that discovery-based instructional practices for evidence-based reasoning are positively associated with students' beliefs about knowledge and knowing.

Moreover, the study discovered that EB positively predicted PPSC. It suggests that higher-order epistemic beliefs are associated with stronger PPSC. Students who hold a belief that knowledge is dynamic and justifiable are likely to employ adaptive reasoning strategies and stick with difficult problems to be solved. This finding is consistent with earlier research undertaken by Xu et al. (2024), in which they revealed that epistemic beliefs can predict students' self-regulated learning behavior and ability to construct effective problem-solving approaches. Moreover, Zhao et al. (2024) and Depaepe et al. (2016) found that students with higher epistemic cognition possess higher metacognitive monitoring levels, where they can evaluate different routes of solutions when confronted with challenging mathematical issues. With reference to the current study, the result validates the conclusion that epistemic beliefs are not only cognitive outputs but also motivate and strategically facilitate mathematical thinking and performance.

Contrary to the hypothesized relationship, the findings revealed that epistemic beliefs did not significantly moderate the relationship between IBL and PPSC. This indicates that the effectiveness of IBL in enhancing students' perceived competence does not depend on their epistemic belief levels. While graphical trends suggested slight variations across groups, these differences were not statistically meaningful and should be interpreted cautiously. This finding suggests that epistemic beliefs function more as an independent predictor of learning outcomes rather than a condition that alters the effectiveness of inquiry-based instruction. Similar patterns have been reported in prior studies where cognitive and epistemic variables exert direct rather than interactive effects on learning outcomes (Schiefer et al., 2022).

From a sustainable education perspective, the findings of this study provide important insights into how mathematics learning can be structured to promote long-term learner development. First, the significant effect of IBL highlights its role in fostering durable mathematical reasoning. By engaging students in exploration, justification, and reflection, inquiry-based instruction supports deeper conceptual understanding that extends beyond immediate classroom tasks and contributes to sustained problem-solving competence over time. Second, the strong direct effect of epistemic beliefs demonstrates their importance for long-term conceptual engagement. Students who perceive mathematical knowledge as dynamic, justifiable, and open to reasoning are more likely to persist in challenging tasks and apply flexible strategies (Boateng et al., 2026). Finally, these findings have important implications for curriculum design and teacher practice in secondary schools. Mathematics instruction should emphasize inquiry, reasoning, and conceptual understanding rather than procedural repetition, while teachers should create environments that encourage questioning, multiple solution pathways, and reflection.

7. Theoretical Implications

This study adds some theoretical implications to mathematics learning. Firstly, the positive relationships among PPSC, IBL, and epistemic beliefs confirm the fundamental assumptions of the constructivist theory of learning in the sense that it assumes students construct knowledge actively by seeking, thinking, and sharing. The study is able to show that inquiry pedagogies encourage cognitive achievement and epistemic development, and it has evidence that students develop an understanding of knowledge in a different way due to learning through problem-solving. That the study reveals such a connection highlights the point that cognitive and epistemic development are complementary facilitative dimensions of learning. Second, the results provide empirical support for epistemic cognition theory, which posits that students' beliefs about knowledge and knowing influence how they process information and engage with learning tasks (Hofer, 2016). The findings show that epistemic beliefs function as a strong direct predictor of PPSC rather than as a moderating variable. This suggests that epistemic beliefs play a foundational role in shaping learners' reasoning and engagement, independent of instructional conditions. The finding contributes to the theories by positioning epistemic beliefs as both a cognitive by-product and a contextual determinant of learning effectiveness. Lastly, the results extend social constructivist theory by illustrating how collective talk and interaction within inquiry classrooms give rise to the co-construction of conceptual knowledge and epistemic awareness. The Ghanaian setting further contributes to theoretical understanding by illustrating that inquiry pedagogy can lead to real cognitive and epistemic change even in examination-based systems. By doing this, the study responds to theory by bringing together epistemic cognition theories as well as IBL theories and putting them in the middle as a powerful predictor of students' perceived mathematical problem-solving competence.

8. Practical Implications for Classroom Practice

The outcomes of this study have significant implications for the learning of mathematics and curriculum implementation, particularly for senior high schools. The positive associations of IBL with PPSC and epistemic beliefs support encouraging teachers of mathematics adopt more student-centered and inquiry-based pedagogies. The activities in the classroom must be planned by the teachers in such a way that questioning, inquiry, and justification, and not blind memorization of procedures, will be encouraged. These pedagogies help in achieving conceptual understanding and students' confidence in solving non-routine mathematical problems. Furthermore, the discovery that epistemic beliefs are crucial in problem-solving competence underpins the necessity to introduce epistemic reflection in mathematics education. For instance, educators can engage learners in arguing for the creation, justification, and utilization of mathematical knowledge in an attempt to evoke higher levels of understanding of conceptions of knowledge. Although the moderating role of epistemic beliefs was not supported, the findings indicate that the IBL-PPSC association did not vary significantly across epistemic belief levels, while epistemic beliefs independently contribute to students' perceived competence in problem-solving. Hence, results encourage a shift in instruction to include the use of inquiry, reflection, and epistemic discourse for mathematics cognitive and affective development.

9. Conclusion

The current study examined relationships among IBL, epistemic beliefs, and PPSC among senior high school students, including the proposed moderating role of EB. IBL significantly predicted both PPSC and EB, and EB significantly predicted PPSC. However, EB did not significantly moderate the relationship between IBL and PPSC. These findings indicate that IBL and epistemic beliefs make independent contributions to students' PPSC. Because the study used cross-sectional self-report data, the findings should be interpreted as predictive associations rather than causal effects. Within a sustainable mathematics education perspective, the results highlight the relevance of inquiry-oriented instruction and productive epistemic beliefs for supporting students' perceived capacity to engage with mathematical problems over time.

10. Limitations and Future Research Directions

This study has several limitations. First, its cross-sectional design does not permit strong causal conclusions about the direction of relationships among IBL, EB, and PPSC. Second, all constructs were measured using self-report data; therefore, PPSC reflects students' perceived competence rather than objectively assessed mathematical performance, and common method variance may have influenced the observed relationships. Third, the sample was drawn from selected senior high schools in one region of Ghana, which may limit generalizability. Future research should use longitudinal or experimental designs, include objective measures of mathematical problem solving, and replicate the model across regions and school contexts. Because the moderation effect was not statistically significant, future studies may also examine alternative mechanisms through which epistemic beliefs relate to engagement and achievement in inquiry-based mathematics classrooms.

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Data availability statement: The data supporting the findings of this research can be obtained from the corresponding author upon request.

Declaration of interest: The authors confirm that there were no conflicts of interest related to the study.

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